Unitary Symmetry and the Transformation $\eta \to \pi^0$*

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The transformation $\eta \to \pi^0$, which is thought to play a dominant role in the decay mode $\eta \to \pi^+\pi^-\pi^0$, is examined in the light of unitary symmetry and several different models for η decay. Beside the models of Gell-Mann, Sharp, and Wagner (GSW), and of Barrett and Barton (BB), these include the strong coupling of pseudoscalar and vector meson octets with the unitary singlet vector meson φ, and the electromagnetic coupling of pseudoscalar and vector mesons. Group-theoretical methods are used to confirm that the contributions of all but one of the lowest order diagrams on the GSW model cancel one another. The same methods show that the cancellations in lowest order are not as serious for the BB model, and are nonexistent for models involving the φ meson.

INTRODUCTION

IN the unitary symmetry scheme of Gell-Mann and Ne'eman,[1] the particles of a given unitary multiplet can be classified not only by means of the isotopic-spin subgroup of $U(3)$, but also by two other $U(2)$ subgroups of $U(3)$.[2–6] These alternative classifications have proved to be very useful tools for predicting relations between strong-interaction processes,[4,5] and for studying the electromagnetic properties of elementary particles.[5,6] Their usefulness can further be illustrated by a study of various models for the process $\eta \to \pi^0$.

Recent experimental data[7] appear to confirm the proposal of Barton and Rosen[8] that the decay $\eta \to \pi^+\pi^-\pi^0$ proceeds mainly through a single-pion intermediate state. While the precise mechanism by which the η transforms into a neutral pion has no effect on the spectrum of the final state, it is of considerable interest if we wish to relate the properties of $\eta \to \pi^+\pi^-\pi^0$ to those of other η decay modes. In their original work, Barton and Rosen make use of the model of Gell-Mann, Sharp, and Wagner,[9] according to which all modes of η decay are dominated by a strong primary dissociation into two virtual vector mesons; the single-pion intermediate state is then reached by means of electromagnetic interactions. Further investigation[10] has revealed, however, that, under the assumption of unitary symmetry, the contributions to the matrix element $\langle\eta|\pi^0\rangle$ from many of the lowest order diagrams cancel one another. Because of this result, it becomes worthwhile to consider the transformation $\eta \to \pi^0$ on the basis of other models for η decay.

One alternative to the GSW model is the model of Barrett and Barton.[11] These authors assume that the η undergoes a strong primary dissociation into baryon-antibaryon pairs; the rate for $\eta \to 2\gamma$ can be calculated in the same way as the rate for $\pi^0 \to 2\gamma$, and $\eta \to \pi^+\pi^-\pi^0$ appears to be dominated by the single pion intermediate state.[12] Another model can be obtained by coupling the octets of pseudoscalar and vector mesons with the unitary singlet vector meson φ.[13] Also, because the quantum numbers of the[14] η require its decay modes to proceed via electromagnetic interactions, it may not be unreasonable to assume that the η undergoes a primary dissociation into a vector meson and a photon.

Here we shall show that the cancellations among the lowest order diagrams of the GSW model can be established by the group theoretical methods mentioned above. It is a relatively simple matter to apply these methods to the other models in order to find out where cancellations occur, and which diagrams can be expected to dominate $\eta \to \pi^0$.

In the next section we show how the various classification schemes for particles in a given unitary multiplet can be obtained, and then derive expressions, in unitary spin space, for electromagnetic and strong interactions. The lowest order diagrams for $\eta \to \pi^0$ are considered in the third section, and the conclusions to be drawn from

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[1] M. Gell-Mann, California Institute of Technology Report CTSL-20, 1961 (unpublished); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

[2] C. A. Levinson, H. J. Lipkin, and S. Meshkov, Nuovo Cimento **23**, 236 (1961).

[3] C. A. Levinson, H. J. Lipkin, and S. Meshkov, Phys. Letters **1**, 44 (1962).

[4] S. Meshkov, C. A. Levinson, and H. J. Lipkin, Phys. Rev. Letters **10**, 361 (1963).

[5] C. A. Levinson, H. J. Lipkin, and S. Meshkov (to be published).

[6] S. P. Rosen, Phys. Rev. Letters **11**, 100 (1963).

[7] D. Berley, D. Colley, and J. Schultz, Phys. Rev. Letters **10**, 114 (1963).

[8] G. Barton and S. P. Rosen, Phys. Rev. Letters **8**, 414 (1962); see also C. Kacser, Phys. Rev. **130**, 355 (1963).

[9] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962). Henceforth, this paper is referred to as GSW in the text.

[10] G. Barton (private communication), who uses the coupling constants predicted by unitary symmetry at the vertices of the appropriate diagrams to show the cancellations. See also H. Shimodaira, Frascati Report, LNF-62/99, 1962 (unpublished).

[11] Barbara Barrett and G. Barton, Phys. Letters **4**, 16 (1963), and Clarendon Laboratory, Oxford 1963 (unpublished). These papers are referred to as BB in the text.

[12] Barbara Barrett and G. Barton (to be published). These calculations are based on electromagnetic mass differences, rather than on the diagrams of Figs. 2(c) and 2(d) below.

[13] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962). Throughout this paper we use φ to denote the unitary singlet vector meson, and ω to denote the $T=0$ member of the octet of vector mesons. The question whether the observed φ, ω are really admixtures of different unitary multiplets [J. J. Sakurai (to be published)] is not relevant here.

[14] The assignment of quantum numbers to the η is reviewed by D. B. Lichtenberg, Stanford Linear Accelerator Center, Report SLAC-13 (1963) (unpublished).

TABLE I. Classification of pseudoscalar mesons as eigenstates of $(\mathbf{L}^2, L_3, Y_L)$.

Y_L	L	Eigenstates
1	$\frac{1}{2}$	$\pi^+, -\bar{K}^0$
0	1	$K^+, \frac{1}{2}(\pi^0+\sqrt{3}\eta), -K^-$
0	0	$\frac{1}{2}(\sqrt{3}\pi^0-\eta)$
-1	$\frac{1}{2}$	K^0, π^-

this analysis are presented in Sec. 4. Useful mathematical identities are discussed in the Appendix.

MATHEMATICAL FORMULATION

It is convenient to adopt Okubo's formalism[15] for unitary symmetry. The generators A_ν^μ of infinitesimal transformations in $U(3)$ obey the commutation rules

$$[A_\beta^\alpha, A_\nu^\mu]=\delta_\beta^\mu A_\nu^\alpha-\delta_\nu^\alpha A_\beta^\mu \tag{1}$$

and the unitary restriction

$$(A_\nu^\mu)^\dagger=A_\mu^\nu. \tag{2}$$

They can be divided into three sets[6]:

$$T_+=-A_1^2,\quad T_-=-A_2^1,\quad T_3=\tfrac{1}{2}(A_2^2-A_1^1),\quad Y_T=A_3^3,$$
with
$$T_\pm=T_1\pm iT_2; \tag{3}$$

$$L_+=-A_1^3,\quad L_-=-A_3^1,\quad L_3=\tfrac{1}{2}(A_3^3-A_1^1),\quad Y_L=A_2^2,$$
with
$$L_\pm=L_1\pm iL_2; \tag{4}$$

$$K_+=-A_2^3,\quad K_-=-A_3^2,\quad K_3=\tfrac{1}{2}(A_3^3-A_2^2),\quad Y_K=A_1^1,$$
with
$$K_\pm=K_1\pm iK_2; \tag{5}$$

each containing an angular-momentum type operator and a corresponding hypercharge. The first set (3) is identified with isotopic spin and the usual hypercharge

$$Y_T=(B+S), \tag{6}$$

where B denotes baryon number and S strangeness. We use only those representations $U(f_1,f_2,f_3)$ of $U(3)$ for which[15,16]

$$A_1^1+A_2^2+A_3^3=f_1+f_2+f_3=0; \tag{7}$$

with the aid of (6) and (7), we can then make the following identifications:

$$\begin{aligned}&Q=-A_1^1,\\ &L_3=\tfrac{1}{2}(Q+Y_T),\qquad Y_L=(Q-Y_T),\\ &K_3=\tfrac{1}{2}(2Y_T-Q),\qquad Y_K=-Q,\end{aligned} \tag{8}$$

where Q is the electric charge operator.

[15] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).
[16] S. Okubo, Phys. Letters 4, 14 (1963). In this paper Okubo points out the equivalence between representations $U(f_1,f_2,f_3)$ of $U(3)$, for which $f_1+f_2+f_3=0$ and representations of $SU(3)$.

The pseudoscalar mesons form a basis for the representation $U(1,0,-1)$ and can be classified as eigenstates of either $(\mathbf{T}^2,T_3,Y_T)$, $(\mathbf{L}^2,L_3,Y_L)$, or $(\mathbf{K}^2,K_3,Y_K)$; the classifications according to the second and third sets of operators are shown in Tables I and II. Corresponding classifications for vector mesons and metastable baryons are obtained from the substitutions

$$f\rightarrow g,\quad f\rightarrow M, \tag{9}$$

where[13]

$$f\equiv\begin{bmatrix}\pi^+\\ \pi^0\\ \pi^-\\ \eta\\ K^+\\ K^0\\ \bar{K}^0\\ K^-\end{bmatrix},\quad g\equiv\begin{bmatrix}\rho^+\\ \rho^0\\ \rho^-\\ \omega\\ K^{*+}\\ K^{*0}\\ \bar{K}^{*0}\\ K^{*-}\end{bmatrix},\quad M\equiv\begin{bmatrix}\Sigma^+\\ \Sigma^0\\ \Sigma^-\\ \Lambda\\ p\\ n\\ \Xi^0\\ \Xi^-\end{bmatrix}. \tag{10}$$

In this notation the electromagnetic currents of baryons and mesons are[17]

$$\begin{aligned}J(M)&=\bar{M}QM\equiv-\bar{M}A_1^1M,\\ J(f)&=\bar{f}Qf\ \equiv-\bar{f}A_1^1f,\\ J(g)&=\bar{g}Qg\ \equiv-\bar{g}A_1^1g.\end{aligned} \tag{11}$$

Each one behaves under the transformations of[16] $SU(3)$ as the R_1^1 component of a traceless tensor R_ν^μ. The current $J(fg)$ describing the electromagnetic interaction of vector mesons with pseudoscalar mesons is assumed to behave in the same way; however, because π^0 is even under charge conjugation while ρ^0 is odd,[14] its form is different from the currents in (11), viz,

$$J(fg)=\tfrac{1}{3}\bar{g}[A_\lambda^1A_1^\lambda+A_1^\lambda A_\lambda^1-\tfrac{2}{3}A:A]f, \tag{12}$$

where

$$A:A\equiv A_\lambda^\mu A_\mu^\lambda=f_1^2+f_2^2+f_3^2+2(f_1-f_3) \tag{13}$$

for a representation $U(f_1,f_2,f_3)$. With the aid of the commutation relations (1) and the identity (see the Appendix),

$$A_\lambda^1A_1^\lambda\equiv\tfrac{1}{2}A:A-\tfrac{3}{2}Q+(\tfrac{1}{4}Q^2-\mathbf{K}^2), \tag{14}$$

$J(fg)$ can be rewritten as

$$J(fg)=\tfrac{1}{3}\bar{g}[\tfrac{1}{3}A:A+\tfrac{1}{2}Q^2-2\mathbf{K}^2]f. \tag{15}$$

This expression leads to the same relations among the

TABLE II. Classification of pseudoscalar mesons as eigenstates of $(\mathbf{K}^2,K_3,Y_K)$.

Y_K	K	Eigenstates
1	$\frac{1}{2}$	$\pi^-, -K^-$
0	1	$K^0, \frac{1}{2}(-\pi^0+\sqrt{3}\eta), \bar{K}^0$
0	0	$\frac{1}{2}(\sqrt{3}\pi^0+\eta)$
-1	$\frac{1}{2}$	$(K^+, -\pi^+)$

[17] Throughout this paper, the space-time structure of interactions is suppressed.

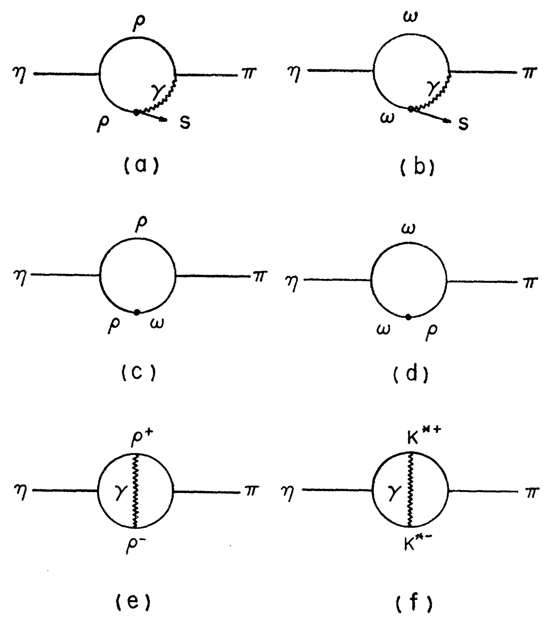

FIG. 1. Lowest-order diagrams for $\eta \to \pi^0$ based on the model of Gell-Mann, Sharp, and Wagner (Ref. 9).

coupling constants for processes $g \to f+\gamma$ as are given by Okubo.[16]

Equations (11) and (15) show clearly that the electromagnetic interactions are invariant under rotations of K spin.[5] This fact can be used to determine the form of the effective current $J(sg)$ corresponding to the transformations

$$\rho \to \gamma, \quad \omega \to \gamma. \tag{16}$$

It can be seen from Table II and Eqs. (9) and (10), that the only K-spin scalar which can be formed from the vector mesons is $\frac{1}{2}(\sqrt{3}\rho^0+\omega)$: hence, $J(sg)$ must be proportional to $\frac{1}{2}(\sqrt{3}\rho^0+\omega)$.

To express this result formally, we introduce a spurion s with the same spin, parity and charge conjugation parity as the π^0. Under the transformations of $SU(3)$, s behaves like an octet with only one nonvanishing component, namely, the K-spin zero component. The effective current for (16) can now be written in the same form as $J(fg)$:

$$J(sg)=\tfrac{1}{3}\bar{g}[\tfrac{1}{3}A:A+\tfrac{1}{2}Q^2-2\mathbf{K}^2]s. \tag{17}$$

The strong interactions of baryons and pseudoscalar mesons are expressed in the usual trilinear form. To obtain their structure in $SU(3)$ space, we note that the outer product of the baryon octet with the antibaryon octet contains two distinct octets, which may be taken as

$$\bar{M}A_\nu{}^\mu M \quad \text{and} \quad \bar{M}[A_\lambda{}^\mu A_\nu{}^\lambda-\tfrac{1}{3}\delta_\nu{}^\mu A:A]M.$$

The most general, $SU(3)$ invariant, interaction will then be a linear combination of the inner products of these octets with the pseudoscalar meson octet.

In the discussion of the next section, we shall need only those terms containing the η and π^0 fields. They are

$$H(\eta M)=\eta\bar{M}[\alpha A_3{}^3+\beta(A_\lambda{}^3A_3{}^\lambda-\tfrac{1}{3}A:A)]M \tag{18}$$

and

$$H(\pi^0 M)=(\pi^0/\sqrt{2})\bar{M}[\alpha(A_1{}^1-A_2{}^2)+\beta(A_\lambda{}^1A_1{}^\lambda-A_\lambda{}^2A_2{}^\lambda)]M. \tag{19}$$

By means of Eqs. (3)–(9) and identities similar to (14) (see the Appendix), these can be rewritten as

$$H(\eta M)=\eta\bar{M}[\alpha Y_T+\beta(\tfrac{1}{6}A:A+\tfrac{3}{2}Y_T+\tfrac{1}{4}Y_T{}^2-\mathbf{T}^2)]M, \tag{20}$$

$$H(\pi^0 M)=(\pi^0/\sqrt{2})\bar{M}[-2\alpha T_3+\beta(-3T_3+\tfrac{1}{2}T_3Y_T+\mathbf{L}^2-\mathbf{K}^2)]M. \tag{21}$$

The strong interactions of pseudoscalar and vector mesons can be treated exactly as above, and the terms we shall need are

$$H(\eta g)=\eta\bar{g}[\gamma Y_T+\delta(\tfrac{1}{6}A:A+\tfrac{3}{2}Y_T+\tfrac{1}{4}Y_T{}^2-\mathbf{T}^2)]g, \tag{22}$$

$$H(\pi^0 g)=(\pi^0/\sqrt{2})\bar{g}[-2\gamma T_3+\delta(-3T_3+\tfrac{1}{2}T_3Y_T+\mathbf{L}^2-\mathbf{K}^2)]g. \tag{23}$$

For convenience, we take the constants α, β, γ, δ to be real, so that the operators standing between $\bar{M}$ and M in (20) and (21) and between $\bar{g}$ and g in (22) and (23) are Hermitian.

THE TRANSFORMATION $\eta \to \pi^0$

The lowest order diagrams for $\eta \to \pi^0$ arising from the GSW model are shown in Fig. (1), and those arising from the BB model are shown in Fig. (2). In the first three diagrams of Fig. (3), the η dissociates into a vector meson and a photon; the second two diagrams arise

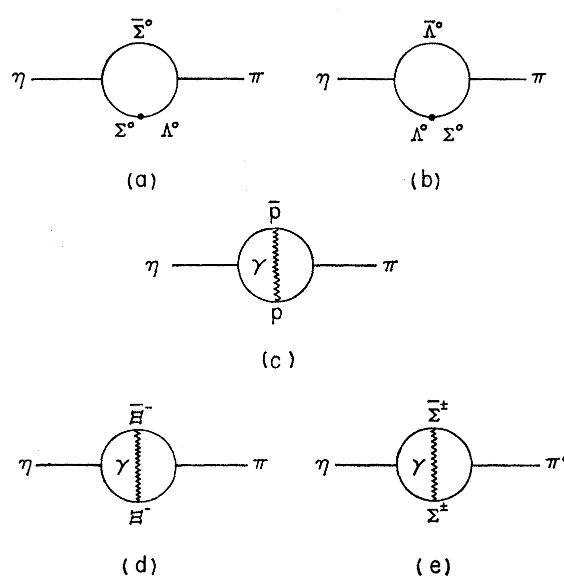

FIG. 2. Lowest-order diagrams for $\eta \to \pi^0$ based on the model of Barrett and Barton (Ref. 11).

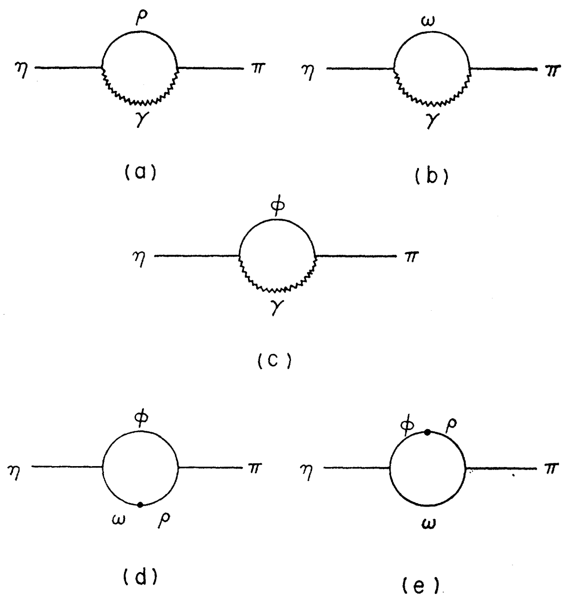

FIG. 3. Lowest-order diagrams for $\eta \to \pi^0$ involving the unitary-singlet vector meson φ.

from the strong coupling of φ with the octets of pseudoscalar and vector mesons.

Diagrams 3(a), 3(b), and 1(a), 1(b)

From the point of view of unitary symmetry, the simplest diagrams to consider are those of Figs. 3(a) and 3(b). The interactions at the η and π^0 vertices are both proportional to $J(fg)$ [Eqs. (12) and (15)], and hence, the sum of the matrix elements for these diagrams is proportional to

$$\langle\eta|J(fg)J(fg)|\pi^0\rangle = \tfrac{1}{9}\langle\eta|[\tfrac{1}{3}A:A+\tfrac{1}{2}Q^2-2\mathbf{K}^2]^2|\pi^0\rangle. \quad (24)$$

Because η and π^0 belong to a basis of the representation $U(1,0,-1)$, we obtain from (13)

$$A:A=6. \quad (25)$$

The matrix element on the right-hand side of (24) now reduces to

$$R_1=\langle\eta|[1-\mathbf{K}^2]^2|\pi^0\rangle. \quad (26)$$

We see from Table II that η and π^0 are the following linear combinations of eigenstates of[18] $\mathbf{K}^2$:

$$\begin{aligned}|\eta\rangle&=\tfrac{1}{2}\{|K=0\rangle+\sqrt{3}|K=1\rangle\},\\ |\pi^0\rangle&=\tfrac{1}{2}\{\sqrt{3}|K=0\rangle-|K=1\rangle\}.\end{aligned} \quad (27)$$

It is now easy to show that

$$R_1=0 \quad (28)$$

and, hence, the contributions of diagrams 3(a) and 3(b) to the matrix element $\langle\eta|\pi^0\rangle$ cancel one another.

[18] The notation $|K=0\rangle$, etc., corresponds to states with $K_3=Y_K=0$; similarly $|T=0\rangle$, etc., are states with $T_3=Y_T=0$.

Consider next diagrams 1(a) and 1(b): the interaction at the π^0 vertex is again $J(fg)$, but at the η vertex it is now $H(\eta g)$ [Eq. (22)]. The interaction for $\rho\to\gamma$ and $\omega\to\gamma$ is proportional to $J(sg)$ in Eq. (17). Notice that we indicate the emission of a spurion in both diagrams. This is done to emphasize the point that the corresponding matrix element is $\langle s|O|\pi^0\rangle$, rather than $\langle\eta|O|\pi^0\rangle$, where O is an operator in $SU(3)$ space. Because of the particular structure of $H(\eta g)$, the only effect of the η vertex is to introduce a factor

$$N(g)=\alpha Y_T+\beta(\tfrac{1}{6}A:A+\tfrac{3}{2}Y_T+\tfrac{1}{4}Y_T^2-\mathbf{T}^2) \quad (29)$$

into O. It now follows that the total matrix element for diagrams 1(a) and 1(b) is

$$R_2=\langle s|J(sg)N(g)J(fg)|\pi^0\rangle. \quad (30)$$

Using Eqs. (15), (17), (25), and (29), we find

$$R_2=(4/9)\langle s|[1-\mathbf{K}^2][\gamma Y_T+\delta(1+\tfrac{3}{2}Y_T+\tfrac{1}{4}Y_T^2-\mathbf{T}^2)]\times[1-\mathbf{K}^2]|\pi^0\rangle. \quad (31)$$

From (27) and the fact s is the $K=0$ member of an octet, it follows that

$$R_2=(2/9)\langle K=0|[\gamma Y_T+\delta(1+\tfrac{3}{2}Y_T+\tfrac{1}{4}Y_T^2-\mathbf{T}^2)]\times\{\sqrt{3}|K=0\rangle+|K=1\rangle\}. \quad (32)$$

The eigenstates of K spin can be re-expressed in terms of eigenstates of isotopic spin with[18] $T=0$, 1 and $T_3=Y_T=0$:

$$\begin{aligned}|K=0\rangle&=\tfrac{1}{2}\{\sqrt{3}|T=1\rangle+|T=0\rangle\},\\ |K=1\rangle&=\tfrac{1}{2}\{-|T=1\rangle+\sqrt{3}|T=0\rangle\}.\end{aligned} \quad (33)$$

Substituting (33) into (32), we find that

$$R_2=0. \quad (34)$$

Therefore the contributions of diagrams 1(a) and 1(b) cancel one another.

Diagrams 1(c), 1(d) and 2(a), 2(b)

If we regard the vertices $\rho\leftrightarrow\omega$ as representing the processes

$$\rho\leftrightarrow\gamma\leftrightarrow\omega, \quad (35)$$

then, from considerations similar to those for diagrams 1(a), 1(b), the total matrix element for diagrams 1(c) and 1(d) is

$$R_3=\langle s|J(sg)N(g)P(g)J(sg)|s\rangle, \quad (36)$$

where

$$P(g)=[-2\gamma T_3+\delta(-3T_3+\tfrac{1}{2}T_3Y_T+\mathbf{L}^2-\mathbf{K}^2)]. \quad (37)$$

Just as the factor $N(g)$ arises at the η vertex because of the structure of $H(\eta g)$, so the factor $P(g)$ arises at the π^0 vertex as a consequence of the structure of $H(\pi^0 g)$ [see Eq. (23)]. From (17), (25), and the fact that $|s\rangle$ is a state with $K=0$, we obtain

$$R_3=(4/9)\langle K=0|N(g)P(g)|K=0\rangle. \quad (38)$$

By means of (33) and the relation

$$[-5T_3Y_T+2(\mathbf{L}^2-\mathbf{K}^2)]|T,T_3,Y_T\rangle = 2\sqrt{3}(1-Q^2)(1-2T)^2|(1-T),T_3,Y_T\rangle \quad (39)$$

for states of unitary octet (see Appendix), it is easy to show that

$$R_3=0. \quad (40)$$

The cancellation of diagrams 1(c) and 1(d) can be shown in a more general fashion. Let us suppose that there exists an effective interaction for $\omega\leftrightarrow\rho$, the origin of which is not relevant here; expressed in terms of the various spin-type operators, it takes the form (see Appendix)

$$H(\omega\rho)=\bar{g}[-5T_3Y_T+2(\mathbf{L}^2-\mathbf{K}^2)]g. \quad (41)$$

Diagrams 1(c) and 1(d) contain closed loops of vector mesons, and therefore the total matrix element for them will be proportional to the trace, in $SU(3)$ space, of the product of the operators at each vertex, viz.,

$$R_4=\mathrm{Tr}(N(g)H(\omega\rho)P(g)) = \sum_{a,b,c}\langle a|N(g)|b\rangle\langle b|H(\omega\rho)|c\rangle\langle c|P(g)|a\rangle, \quad (42)$$

where a, b, c denote states of the vector meson octet. The only nonvanishing matrix elements of $H(\omega\rho)$ are,[18] from (39),

$$\langle T=0|H(\omega\rho)|T=1\rangle=\langle T=1|H(\omega\rho)|T=0\rangle=2\sqrt{3}. \quad (43)$$

Equations (29), (37), (39), and (43) can then be used to show that

$$R_4=0. \quad (44)$$

Since Σ^0 and Λ^0 behave, under the transformations of $SU(3)$, in exactly the same way as ρ^0 and ω, respectively, the same argument can be used to show that the contributions of diagrams 2(a) and 2(b) to $\langle\eta|\pi^0\rangle$ cancel one another.

Diagrams 1(e), 1(f) and 2(c), 2(d), 2(e)

To evaluate the contributions of diagrams 1(e) and 1(f) to $\langle\eta|\pi^0\rangle$, we note that they also contain closed loops of vector mesons. Hence the sum of their matrix elements is proportional to

$$R_5=\mathrm{Tr}[N(g)J(g)P(g)J(g)]. \quad (45)$$

From the expression for $J(g)$ in Eq. (11), we obtain

$$R_5=\sum_{a,b}Q_aQ_b\langle a|N(g)|b\rangle\langle b|P(g)|a\rangle, \quad (46)$$

where Q_a and Q_b denote the electric charges of the states a and b of the vector-meson octet. From (25) and (29), we obtain

$$\begin{aligned}\langle\rho^\pm|N(g)|\rho^\pm\rangle&=-\delta,\\ \langle K^{*\pm}|N(g)|K^{*\pm}\rangle&=\pm\gamma+\tfrac{1}{2}\delta(1\pm3),\\ \langle\rho^\pm|N(g)|\rho^\mp\rangle&=\langle K^{*\pm}|N(g)|K^{*\mp}\rangle=0;\end{aligned} \quad (47)$$

similarly, from (37) and (39), we obtain

$$\begin{aligned}\langle\rho^\pm|P(g)|\rho^\pm\rangle&=\mp(2\gamma+3\delta),\\ \langle K^{*\pm}|P(g)|K^{*\pm}\rangle&=\mp\gamma+\tfrac{1}{2}\delta(3\mp3).\end{aligned} \quad (48)$$

Equations (47) and (48) show that the contribution to R_5 from ρ-meson states is zero. To understand this result, we note that in diagram 1(e) the ρ mesons are emitted at the η vertex in a pure $T=0$ state; the exchange of a photon does not alter this state, and hence the ρ mesons cannot annihilate to form a $T=1$ π meson.

It is also apparent from (47) and (48) that the contributions from K^*-meson states do not vanish for arbitrary values of γ, δ. They will only do so when the nature of the strong interaction between pseudoscalar and vector mesons, as determined by the coupling constants γ, δ, in (22) and (23), is such as to cause an accidental cancellation.

The same analysis can be used to show that, while the diagram 2(e) gives no contribution to $\langle\eta|\pi^0\rangle$, the contributions from 2(c) and 2(d) will only cancel each other for a particular choice of the $SU(3)$ invariant strong interactions of pseudoscalar mesons and baryons.

Diagrams 3(c), 3(d), and 3(e)

If the unitary octets of pseudoscalar and vector mesons are strongly coupled to the unitary singlet vector meson φ,[13] then the transformation $\eta\to\pi^0$ can occur as in diagrams 3(d) and 3(e). These diagrams have different structures in $SU(3)$ space, and, therefore, they cannot cancel one another for reasons of unitary symmetry alone.

The lowest order diagram involving the dissociation of the η into the φ vector meson and a photon is shown in Fig. 3(c). Since there are no other diagrams of the same order, there will be no cancellations.

CONCLUSIONS

The preceding discussion shows that, as a consequence of unitary symmetry, the only diagrams which can contribute to $\langle\eta|\pi^0\rangle$ are:

(a) Figure 1(f) for the GSW model[9];
(b) Figures 2(c) and 2(d) for the BB model[11];
(c) Figures 3(d) and 3(e) for the model based on the strong interactions of the unitary singlet φ[13]; and
(d) Figure 3(c) for the model based on the electromagnetic coupling of pseudoscalar mesons with φ.

The most serious cancellation occurs in the GSW model, where only one out of six diagrams survives. In the BB model, two of the five possible diagrams do not cancel, and in the φ-meson models there are no cancellations in lowest order.

To make a choice between these models, it will be necessary first to find out how seriously the above results are disturbed by the strong interactions that violate unitary symmetry; and secondly, to compare their predictions for η decay modes other than $\eta \longrightarrow \pi^0 \longrightarrow \pi^+\pi^-\pi^0$.

ACKNOWLEDGMENT

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APPENDIX

(i) The following identity can be proved by means of the commutation relations in Eq. (1):

$$4A_\lambda{}^1A_1{}^\lambda \equiv 2A:A+2(A_1{}^1)^2+2(3A_1{}^1-A_\mu{}^\mu)-2A_3{}^2A_2{}^3 -2A_2{}^3A_3{}^2-(A_3{}^3-A_2{}^2)^2-(A_1{}^1-A_\mu{}^\mu)^2. \quad (49)$$

From (5), (7), and (8), it then follows that

$$A_\lambda{}^1A_1{}^\lambda \equiv \tfrac{1}{2}A:A-\tfrac{3}{2}Q+\tfrac{1}{4}Q^2-\mathbf{K}^2. \quad (50)$$

Similarly,

$$A_\lambda{}^2A_2{}^\lambda \equiv \tfrac{1}{2}\boldsymbol{A}:A+\tfrac{3}{2}Y_L+\tfrac{1}{4}Y_L{}^2-\mathbf{L}^2. \quad (51)$$

The identities in (50) and (51) are the analogs of Okubo's identity[15] for $A_\lambda{}^3A_3{}^\lambda$.

(ii) Equation (39) is an *ad hoc* result which applies to the states forming a basis for the representation $U(1, 0, -1)$ of $U(3)$. It can be verified with the aid of Tables I and II, but the author has not found a proof for it.

(iii) For reasons of charge conjugation invariance, we require $H(\omega\rho)$ to be of the form[17]

$$\bar{\rho}^0\omega+\bar{\omega}\rho^0.$$

Equation (41) is then a simple consequence of Eq. (39).

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Theory of Spin-$\frac{1}{2}$ Particles with Parity-Nonconserving Interactions*

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It is shown that when interactions are not invariant under parity conjugation, both the self-mass term $(-\delta m\bar{\psi}\psi)$ and the term $(-a\bar{\psi}\gamma_\mu\gamma_5\partial\psi/\partial x_\mu)$ are induced by the self-interaction of any spin-$\frac{1}{2}$ particle with nonvanishing mass. (For simplicity T invariance is assumed.) When $a^2>1$, the spin-$\frac{1}{2}$ particle propagates in vacuum faster than the velocity of light. When $a^2=1$, the observed mass should be zero. Therefore, it follows that $a^2<1$ for any spin-$\frac{1}{2}$ particle with nonvanishing mass. Since $1>a^2>\frac{1}{3}$ implies the existence of ghost states, one must require $a^2\leqslant\frac{1}{3}$. Although a has no physical meaning for free particles, as an example, it is also shown that it has a physical meaning when a charged particle is interacting with an external electromagnetic field. The value of a is estimated for the electron and the muon.

1. INTRODUCTION AND SUMMARY

THE purpose of this work is to study the properties spin-$\frac{1}{2}$ particles possess as a result of parity-nonconserving interactions. To outline our discussions given here, we shall tentatively start from the Lagrangian density

$$L=L_1+L_2, \quad L_1=-\bar{\psi}(x)\left[\gamma_\mu\frac{\partial}{\partial x_\mu}+m_0\right]\psi(x), \quad (1)$$

for a spin-$\frac{1}{2}$ field ψ with mechanical mass m_0, where L_2 is not invariant under C or P transformation but is invariant under CP (or T) transformation. For simplicity we shall consider only CP-invariant interactions throughout this paper.

Since the free particle is interacting with its self-field, L_1 does not express the free part of the Lagrangian density for the dressed spin-$\frac{1}{2}$ field considered. When all interactions are renormalizable and invariant under both C and P transformations, as is well known, $(L_1-\delta m\bar{\psi}\psi)$ is the free part of the Lagrangian density for the dressed particle, where δm is the self-energy of the particle. In our more general case it will be shown in Sec. 2 that, in addition to the self-mass term, the self-interaction induces another term $(-a\bar{\psi}\gamma_\mu\gamma_5\cdot\partial\psi/\partial x_\mu)$, where $\gamma_5{}^2=1$ and a is a real constant. This term should be added to the free part of the Lagrangian density and consequently be subtracted from L_2, as the self-mass term is, to perform the renormalization consistently.

To discuss the magnitude of the coefficient a of the parity-nonconserving counter term, consider the Lagrangian density

$$-\bar{\psi}(x)\left[\gamma_\mu(1+a\gamma_5)\frac{\partial}{\partial x_\mu}+\mu\right]\psi(x) \quad (2)$$

or the Dirac equation

$$[i\gamma p(1+a\gamma_5)+\mu]\psi(p)=0 \quad (3)$$

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